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Effects of the upper level coupling field on lasing without inversion in a V-type system

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Abstract. Driven by one upper level coupling field, a three-level V-type atomic system with a pair of upper levels is studied. With one strong coupling field and one weak probe field, it is found that, due to the effects of the upper level coupling field, the quantum coherence between the two upper levels can be induced, and the absorption of the probing field is very sensitive to the relative phase of the probe, the pumping and the upper level coupling fields. With proper parameters, lasing without inversion (LWI) can be realized.

PACS. 42.50.-p Quantum optics – 42.50.Nn Quantum optical phenomena in absorbing, dispersive and conducting media – 42.50.Ct Quantum description of interaction of light and matter; related experiments

1 Introduction

In the past decade, much attention has been shown to the quantum coherence or interference between two neighboring spontaneous decay channels [1–4]. The original paper on this spontaneously generated coherence (SGC) effect is reported by Zhu et al. [5]. In their experiment, it can lead to the spectral line elimination and spontaneous emission cancellation. SGC can lead to many interesting phenomena, such as canceling, narrowing, and phase controlling the spontaneous emission [2–9]. In a three-level V-type atomic system with two near degenerate excited levels, we found that lasing without inversion (LWI) and phase controlling group velocity can be realized due to the presence of SGC [10, 11].

However, there are many other papers on quantum coherent effects of the microwave field. For example, by coupling the excited states to each other with a strong microwave field, the quantum coherence is created [12]. With a magnetic field, the very narrow absorption resonances due to the induced coherence is reported [13]. Meanwhile, an inversionless gain is reported in a Λ atomic system driven by a microwave [14]. Recently, by changing the intensity of an external microwave field, the propagation of light can be switched from subluminal to superluminal [15].

Based on our previous paper [10] and noting that a microwave field can generate quantum coherence on a dipole-forbidden transition with two closely lying levels [16–18], we propose a scheme: a three-level V-type atomic system with one external magnetic field, we call it upper level (UL) field, on the two-upper excited levels. Due to the presence of the UL field, the absorption properties of the probe field is strongly dependent on the relative phase of the probe, the pumping and the UL fields. This scheme is different from references [10, 19] and references therein. It is shown that by modulating UL field and the relative phase of the three fields, the gain property of the probe is changed and can be controlled. LWI can be exhibited without the incoherent pumping field [19] and SGC effects [10].

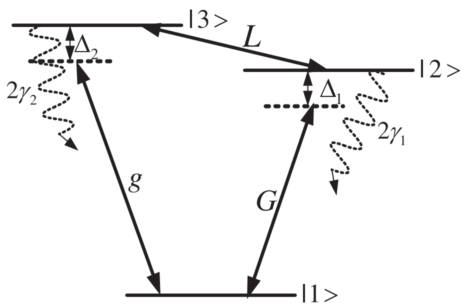

Fig. 1. Schematic diagram of three level V-type system with an additional field L.

2 The atomic model and its basic equations

As shown in Figure 1, we consider a closed three-level V-type atomic system. Transitions $|1\rangle \leftrightarrow |3\rangle$ and $|1\rangle \leftrightarrow |2\rangle$ are assumed to be dipole allowed [14, 15]. A coupling field

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(ω_c) with Rabi frequency G drives transition $|1\rangle \leftrightarrow |2\rangle$. A weak field of frequency ω_p and Rabi frequency g is used to probe the gain-absorption on transition $|1\rangle \leftrightarrow |3\rangle$. The transition $|2\rangle \leftrightarrow |3\rangle$ is generally an electric dipole forbidden while a magnetic dipole allowed transition. Levels $|2\rangle$ and $|3\rangle$ are metastable states. An additional coupling field, referred to as upper level (UL) field, acts on the transition $|2\rangle \leftrightarrow |3\rangle$. The application of UL field can produce regions in the optical response with an appropriate absorption-dispersion profile. The nature of UL field depends on the level structure of the atom. It may be a microwave fields, an infrared field. Moreover, it could be a dc field if one is considering transparency with Zeeman sublevels [14,15]. $2\gamma_i$ $(i = 1, 2)$ is the spontaneous decay rate from two upper levels to level $|1\rangle$.

In this paper, we assume that the two excited levels $|2\rangle$ and $|3\rangle$ are closely spaced. Make the dipole moments of the two transitions orthogonal, and then no SGC exists, which is different from our previous work [10]. Here, by coupling the two excited states of the V system with a UL field, the quantum coherence is achieved. Therefore, this scheme can be extended to the case that the large separation is established between the two upper excited states.

The three fields interacting with the V-type system are as the following

$$\tilde{\boldsymbol{E}}_{p,c} = \boldsymbol{E}_{p,c} e^{-i(\omega_{p,c}t+\phi_{p,c})} + c.c.,$$
$$\tilde{\boldsymbol{B}} = \boldsymbol{B} e^{-(i\omega_b t+\phi_b)} + c.c., \tag{1}$$

where ϕ_p, ϕ_c and ϕ_b are the phases of the probe, the coupling and the UL fields, respectively. For simplicity, assume the frequencies satisfy $\omega_p = \omega_c + \omega_b$.

Under the rotating-wave and the electric-dipole approximations, the equations of the motion for the density matrix are given by

$$\begin{aligned}
\dot{\rho}_{12} &= -(\gamma_1 + \Gamma_{12} + i\Delta_1)\rho_{12} + iG(\rho_{22} - \rho_{11}) \\
&\quad + ige^{i\phi}\rho_{32} + iLe^{i\phi}\rho_{13}, \\
\dot{\rho}_{13} &= -(\gamma_2 + \Gamma_{13} + i\Delta_2)\rho_{13} + ig(\rho_{33} - \rho_{11}) \\
&\quad + iGe^{-i\phi}\rho_{23} + iLe^{-i\phi}\rho_{12}, \\
\dot{\rho}_{23} &= -[(\gamma_1 + \gamma_2 + \Gamma_{23}) - i(\Delta_1 - \Delta_2)]\rho_{23} \\
&\quad + iGe^{i\phi}\rho_{13} - ige^{i\phi}\rho_{21} + iL(\rho_{33} - \rho_{22}), \\
\dot{\rho}_{33} &= -2\gamma_2\rho_{33} + ig(\rho_{13} - \rho_{31}) + iL(\rho_{32} - \rho_{23}), \\
\dot{\rho}_{22} &= -2\gamma_1\rho_{22} + iG(\rho_{12} - \rho_{21}) + iL(\rho_{23} - \rho_{32}),
\end{aligned} \tag{2}$$

constrained by the relation $\rho_{11} + \rho_{22} + \rho_{33} = 1$ and $\rho_{ij} = \rho_{ji}^*$. $L = (\boldsymbol{\mu}_{23} \cdot \boldsymbol{B})/(2\hbar)$ is the Lamor frequency of the UL field [13,15]. For simplicity, $\Omega = \boldsymbol{d}_{12} \cdot \boldsymbol{E}_c/(2\hbar)$ and $g = \boldsymbol{d}_{13} \cdot \boldsymbol{E}_p/(2\hbar)$ are chosen to be real, which are Rabi frequencies of the coupling and the probe fields associated with the transitions $|3\rangle \leftrightarrow |1\rangle$ and $|2\rangle \leftrightarrow |1\rangle$. The electric and magnetic dipole-matrix element are represented by $\boldsymbol{d}_{ij}$ and $\boldsymbol{\mu}_{ij}$, respectively. Note that magnetic dipoles are much weaker than electric dipoles [15]. Detunings Δ_1 and Δ_2 are, respectively, defined by $\Delta_1 = \omega_{21} - \omega_c$ and $\Delta_2 = \omega_{31} - \omega_p$. The phase difference among the three fields is $\phi = (\omega_c + \omega_b - \omega_p)t + (\phi_c + \phi_b - \phi_p)$. Therefore, the relative phase is time-dependent, and relevant to both the frequency and the initial phase of the applied field. Γ_{12}, Γ_{13} and Γ_{23} denote the collision dephasing rates.

3 Analysis of the UL field effects

For simplicity, assume that the applied fields is satisfied the condition $\omega_c + \omega_b - \omega_p = 0$. Then the relative phase becomes $\phi = \phi_c + \phi_b - \phi_p$, i.e., it is only relevant to the initial phase of the applied fields.

As we know, the gain-absorption coefficient on transition $|1\rangle \leftrightarrow |3\rangle$ is proportional to the imaginary part of ρ_{13}, and the probe gain will be obtained if $\mathrm{Im}(\rho_{13}) > 0$. Here, the steady-state solution of $\mathrm{Im}(\rho_{13})$ is so complicated since it depends on many parameters. The probe field exhibits amplification when we choose the parameters appropriately, and it is necessary to give the explicit conditions for LWI. For this purpose, first of all, we give the analytical steady-state solutions of equation (2) with $\gamma_1 = \gamma_2 = \gamma$ and $\Delta_1 = \Delta_2 = 0$, where we obtain the analytical solutions in the steady state and derive the condition LWI occurring. Then, $\Delta_1 \neq \Delta_2$ is considered. The numerical simulation will demonstrate that the contributions of the different parts to the probe gain are different: the probe laser can sometimes be amplified, while sometimes attenuated.

According to equation (2), under the condition of two-photon resonance $\Delta_1 = \Delta_2 = 0$, the following can be obtained

$$\begin{aligned}
\rho_{33} - \rho_{22} &= \frac{2}{D}[g^2 + G^2 + (1 + L^2)(2 + \Gamma_{23})][(g^2 - G^2) \\
&\quad \times (2 + g^2 + G^2 - 2L^2 + \Gamma_{23}) + 2gGL(4 \\
&\quad + \Gamma_{23})\sin\phi], \\
\rho_{33} - \rho_{11} &= -\frac{2}{D}[g^2 + G^2 + (1 + L^2)(2 + \Gamma_{23})][G^4 + g^2 \\
&\quad \times (1 + G^2 - L^2) + G^2(3 - 3L^2 + \Gamma_{23}) + (1 \\
&\quad + L^2)(2 + 2L^2 + \Gamma_{23}) - gGL(4 + \Gamma_{23})\sin\phi], \\
\mathrm{Im}[\rho_{13}] &= -\frac{1}{D}\{g[2g^4 + 2G^4(1 + \Gamma_{23}) + 2(1 + L^2)(2 \\
&\quad + \Gamma_{23})(2 + 2L^2 + \Gamma_{23}) + G^2(2 + \Gamma_{23})(L^2(\Gamma_{23} \\
&\quad - 4) + 2(2 + \Gamma_{23})) + 2g^2(2(2 + \Gamma_{23}) + G^2(2 \\
&\quad + \Gamma_{23}) + L^2(4 + \Gamma_{23}))] - GL[gGL(4 + \Gamma_{23})^2 \\
&\quad \times \cos(2\phi) + 2(g^2 + G^2 - 2 - 2L^2 - \Gamma_{23})(g^2 \\
&\quad + G^2 + (1 + L^2)(2 + \Gamma_{23}))\sin\phi]\},
\end{aligned} \tag{3}$$

with

$$\begin{aligned}
D &= 4g^6 + 2g^4[9 + 4\Gamma_{23} + 3G^2(2 + \gamma_{23}) + L^2(5 + 2\Gamma_{23})] \\
&\quad + 2[G^2 + (1 + L^2)(2 + \Gamma_{23})][2G^4 + (1 + L^2)(2 + 2L^2 \\
&\quad + \Gamma_{23}) + G^2(5 + L^2 + 2\Gamma_{23})] + g^2\{6G^4(2 + \Gamma_{23}) + 2 \\
&\quad \times (1 + L^2)(2(2 + \Gamma_{23})(3 + \Gamma_{23}) + L^2(4 + \Gamma_{23})) + G^2 \\
&\quad \times [36 + 28\Gamma_{23} + 6\Gamma_{23}^2 + L^2(\Gamma_{23}(3\Gamma_{23} - 4) - 28)]\} \\
&\quad - 3g^2G^2L^2(4 + \Gamma_{23})^2\cos(2\phi).
\end{aligned} \tag{4}$$

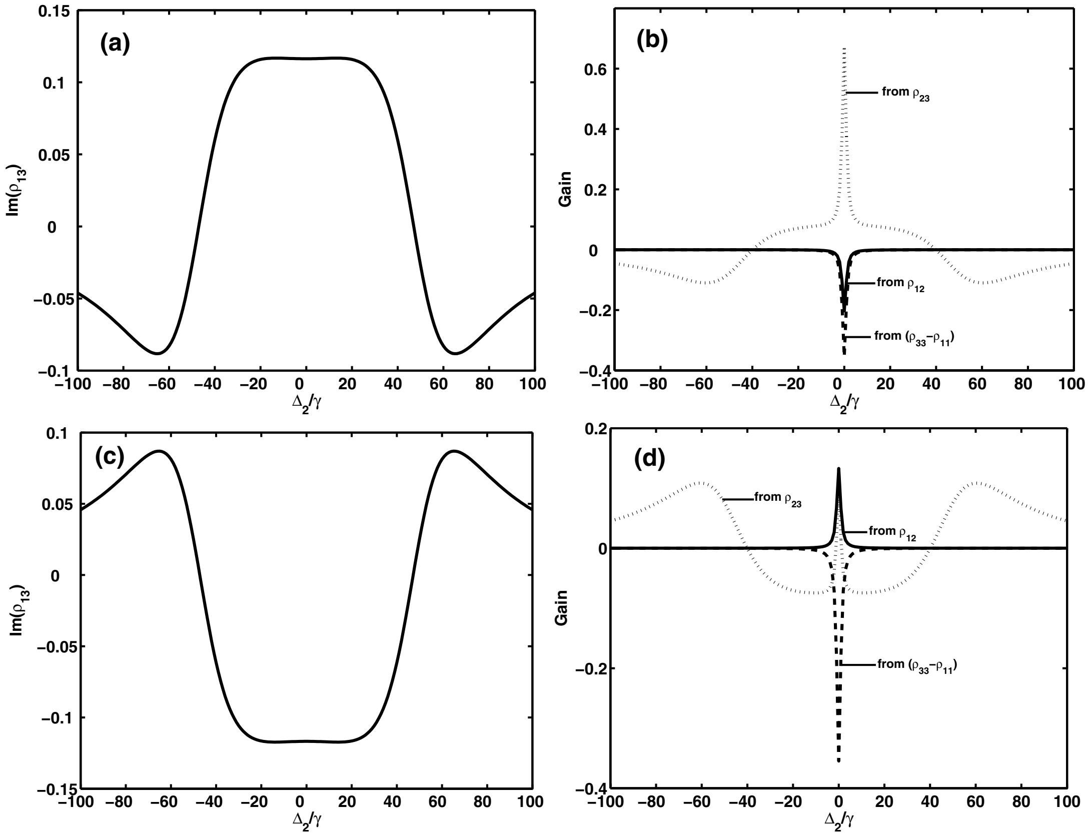

Fig. 2. Spectra for $\phi = \pi/2$ and $\phi = 3\pi/2$. When $\phi = \pi/2$, the curves for (a) the amplification line and (b) the amplification contributions from the population inversion term (dashed line); from ρ_{23} (dotted line); and from ρ_{12} (solid line). The curves for the cases with $\phi = 3\pi/2$ are plotted in (c) and (d). $g = \gamma$, $G = 50\gamma$, $L = 15\gamma$ and $\Gamma_{23} = 0.01\gamma$.

Note, in this paper, the parameters are reduced to dimensionless units by scaling with γ. Here, $\Gamma_{12} = \Gamma_{13} = 0$ for there are much smaller than the decay rate γ in general case. According to equation (3), $\mathrm{Im}(\rho_{13}) \neq 0$ even when $g = 0$ for the contribution of the second term. Therefore, the polarization does not vanish and will induce the generation of the probe field. So, the generation of the probe field is not only due to itself, but also to the relative phase, the coupling and the UL fields.

LWI requires that $\rho_{33} < \rho_{11}$, $\rho_{33} < \rho_{22}$ and $\mathrm{Im}(\rho_{13}) > 0$ [10,19,20]. In the case of $G, L \gg g, \gamma$, the conditions for gain without population inversion can be derived from equation (3) as following

$$G^2 + \Gamma_{23} > 2L^2, \ \sin\phi > 0. \tag{5}$$

The first inequality of equation (5) is the inversionless condition, and the second is the gain condition. When the intensities of the pumping and the external UL fields, and the relative phase satisfy the above inequalities, the amplification of the probe field without population inversion will exhibit.

According to equation (2), in the steady state, ρ_{13} can be written as

$$\rho_{13} = i\frac{g(\rho_{33} - \rho_{11}) + Ge^{-i\phi}\rho_{23} + Le^{-i\phi}\rho_{12}}{\gamma_2 + i\Delta_2}. \tag{6}$$

Here, the probe gain is contributed by three terms, they are: the population difference $(\rho_{33} - \rho_{11})$, two photon coherence ρ_{23} (influenced by the UL field), and the pumping field (couples the states $|1\rangle$ and $|2\rangle$). Comparing equation (6) with equation (12) in [10], it shows that due to the presence of the UL field, the SGC term has be substituted, and ρ_{13} is very sensitive to the intensity of UL field and the relative phase ϕ. This makes LWI possible.

In Figure 2, the gain line and the separate contributions from the three terms of equation (6) are plotted versus detuning Δ_2 with $\Delta_1 = 0$. $\phi = \pi/2$ and $3\pi/2$ are plotted in Figures 2a–2d, respectively. Comparing Figures 2a and 2c, it is shown that LWI occurs in different regions for different ϕ. When $\phi = \pi/2$, the area is near resonant with atomic transition frequency and is symmetrical. When $\phi = 3\pi/2$, the gain region is far away from the center and it is still symmetrical. The contributions to $\mathrm{Im}\rho_{13}$ are plotted in Figures 2b and 2d. When $\phi = \pi/2$, the gain gain is solely generated by ρ_{23} (related with G and ϕ), for the positive contribution from this term is much larger than the negative contributions from $(\rho_{33} - \rho_{11})$ (related with g) and ρ_{12} (related with L and ϕ). However, when ϕ changes to $3\pi/2$ (shown in Fig. 2d), the case is quite different. The probe field is attenuated because the positive contribution from ρ_{12} is much smaller than the negative contribution from $(\rho_{33}-\rho_{11})$. Meanwhile, Raman

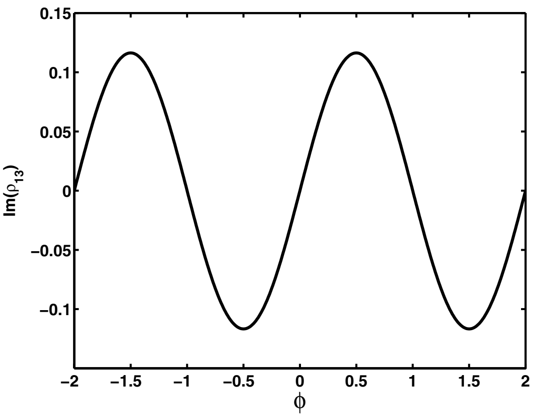

Fig. 3. $\mathrm{Im}(\rho_{31})$ as a function of the relative phase ϕ. $\Delta_1 = \Delta_2 = 0$. Other parameters are the same as those in Figure 2.

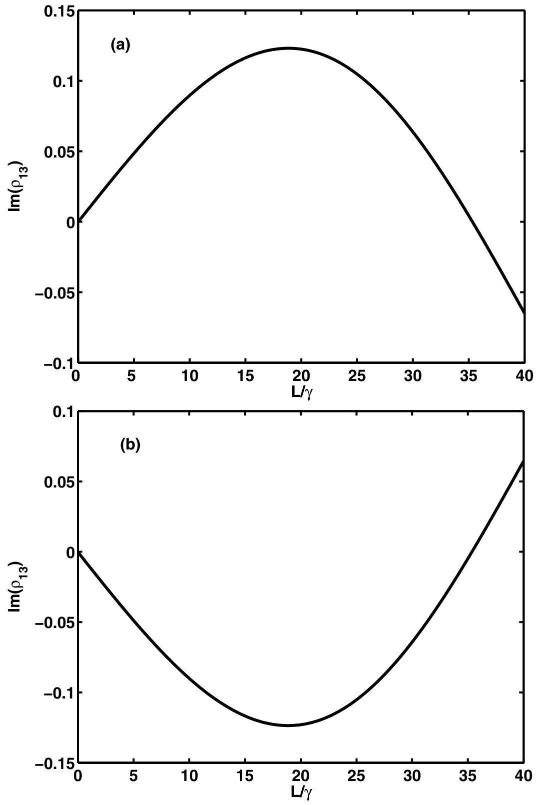

Fig. 4. $\mathrm{Im}(\rho_{31})$ as a function of the UL field L. (a) $\phi = \pi/2$; (b) $\phi = 3\pi/2$. Other parameters are the same as those in Figure 3.

inversion does not occur, and there is no contribution of Raman gain to the probe field. So, due to the effects of the UL field, a three-level V-type system can exhibit LWI without incoherent pumping or the SGC effects, which differs from the conclusions drawn previously [9,18,20]. The probe amplification is the result of two-photon coherence, which is strongly dependent on the relative phase.

In fact, when the coupling field is strong and resonant ($\Delta_1 = \Delta_2 = 0$), it creates mixing of states $|1\rangle$, $|2\rangle$ and $|3\rangle$. The eigenvalues are $E_\pm = 0$, $\pm\sqrt{G^2+L^2}$ [21, 22]. When $\phi = \pi/2$, the corresponding gain region is

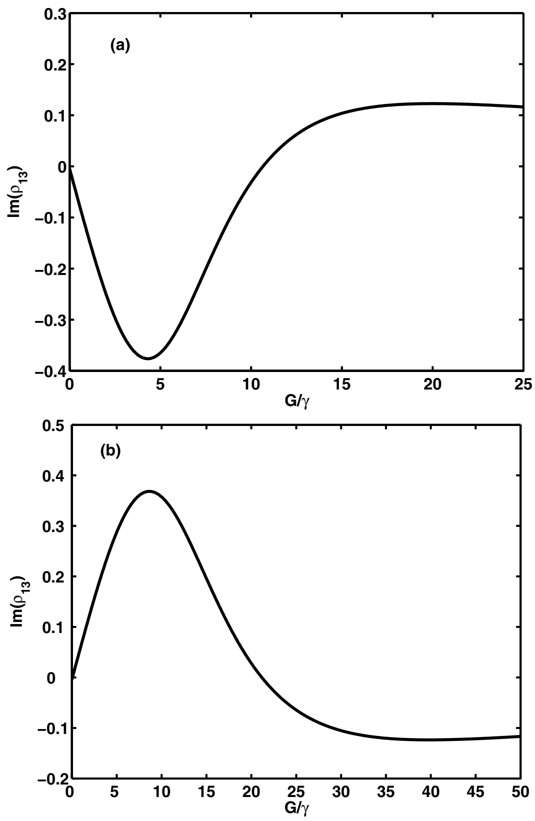

Fig. 5. $\mathrm{Im}(\rho_{31})$ as a function of the coupling field G. (a) $\phi = \pi/2$; (b) $\phi = 3\pi/2$. Other parameters are the same as those in Figure 3.

$|\Delta_2/\gamma| < \sqrt{G^2+L^2}$. Near $|\Delta_2/\gamma| \approx \sqrt{G^2+L^2}$, the absorption is zero, that is electromagnetical induced transparency (EIT). When $\phi = 3\pi/2$, the region for EIT is not changed, but the corresponding gain region is $|\Delta_2/\gamma| > \sqrt{G^2+L^2}$. Varying the value of the UL field and the relative phase, the region where EIT presents will be changed (not shown here).

To gain a deeper insight into the relative phase, the probe gain versus the relative phase is plotted in Figure 3. The other parameters are chosen the same as those in Figure 2. From this figure, it can be seen clearly that the period of the probe amplification is 2π, and it can be observed when $\sin\phi > 0$. When $\sin\phi < 0$, the probe field is absorbed. Therefore, large amplification can be obtained by choosing the relative phase appropriately. These are consistent with the results obtained from Figure 2 and equation (5).

The effects of the UL field and the coupling field are plotted in Figures 4 and 5. From Figure 4, it is shown that when there is no UL field, i.e., $L = 0$, there is $\mathrm{Im}(\rho_{13}) = 0$. That is, EIT is presented. However, when UL field is considered, the situation is changed. From Figure 4a ($\phi = \pi/2$), we see that the value of $\mathrm{Im}(\rho_{13})$ firstly positive and increases, then decreases with the enhancement of the UL field to negative. There exists a L value which makes the probe amplification reach the maximum.

To the case of $\phi = 3\pi/2$, the results are on the contrary (as shown in Fig. 4b). It is obvious that the existence of UL field is the necessary condition for the occurrence of the amplification of the probe field. At the same time, it can be a knob which can be used to change light propagation from absorption to amplification. For the case of Figure 5, it is shown that when there is no pumping field G, EIT is presented. From Figure 5a ($\phi = \pi/2$), we see that the value of $\mathrm{Im}(\rho_{13})$ firstly negative and decreases, then increases with the enhancement of the UL field to positive. There exists a L value which makes the probe amplification reach the minimum. To the case of $\phi = 3\pi/2$, the results are on the contrary (as shown in Fig. 5b). At the same time, G can be a knob which can be used to change light propagation from absorption to amplification. Comparing Figures 4 and 5, the effects of the UL field and the coupling field are similar.

4 Conclusions

In summary, we have shown that, in a V-type atomic system, LWI can be achieved which is related to the additional coupling field, in which the two upper electric-dipole-forbidden states are coupled by a UL field, which induce a magnetic dipole transition. Due to the presence of the UL field, LWI can be obtained by appropriately choosing the Rabi frequency of the pumping field, the Lamor frequency of the UL field and the relative phase between them. It should be emphasized that LWI is achieved in the absence of the incoherent pumping or SGC. Moreover, this scheme is very convenient and much feasible to realize in the experiment than the realization of SGC [10].

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